

Quantum Thermodynamics
 Conference QTD2020
 (virtually in) Barcelona
 21 Oct 2020

Versatile three-dimensional quantum spin dynamics
 equation with guaranteed fluctuation-dissipation link
 Anders, Sait, Horsley, arxiv 2009.00600v1 (2020)

Deriving a generalised Landau–Lifschitz–Gilbert (LLG) equation from a system+bath Hamiltonian

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 University of Exeter (part-time)
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joint work with:
 Simon Horsley
 Connor Sait

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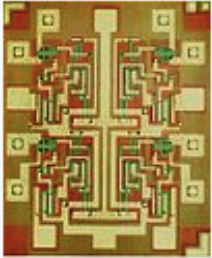
 Universität
 Potsdam

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 EPSRC
 Engineering and Physical Sciences
 Research Council

MOTIVATION

Semiconductor device fabrication



MOSFET scaling (process nodes)

10 μm – 1971

6 μm – 1974

3 μm – 1977

1.5 μm – 1981

1 μm – 1984

800 nm – 1987

600 nm – 1990

350 nm – 1993

250 nm – 1996

180 nm – 1999

130 nm – 2001

90 nm – 2003

65 nm – 2005

45 nm – 2007

32 nm – 2009

22 nm – 2012

14 nm – 2014

10 nm – 2016

7 nm – 2018

5 nm – 2020

Future

3 nm – ~2022

2 nm – ~2023<

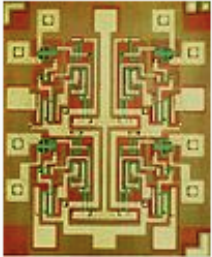


Quantum Thermodynamics understanding will underpin future technologies at the nanoscale



MOTIVATION

Semiconductor device fabrication



MOSFET scaling (process nodes)

| | |
|-------------------|----------|
| 10 μm | – 1971 |
| 6 μm | – 1974 |
| 3 μm | – 1977 |
| 1.5 μm | – 1981 |
| 1 μm | – 1984 |
| 800 nm | – 1987 |
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| Future | |
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Quantum Thermodynamics understanding will underpin future technologies at the nanoscale



In magnetism, the LLG equation has been used for 50 years+, but it is hitting its limits for some materials at short times

Previous LLG extensions have struggled with how to consistently maintain the Fluct-Diss-Rel (FDR), and how to systematically go beyond LLG.



Article | Published: 28 September 2020

Inertial spin dynamics in ferromagnets

Kumar Neeraj, Nilesh Awari, Sergey Kovalev, Debanjan Polley, Nanna Zhou Hag

Nature Physics (2020) | Cite this article

1942 Accesses | 122 Altmetric | Metrics



WHAT IS THE LLG EQUATION?

phenomenological equation that describes the damped dynamics of a spin \mathbf{S} (implicit eq.)

Gilbert damping,
with damping η
(+)

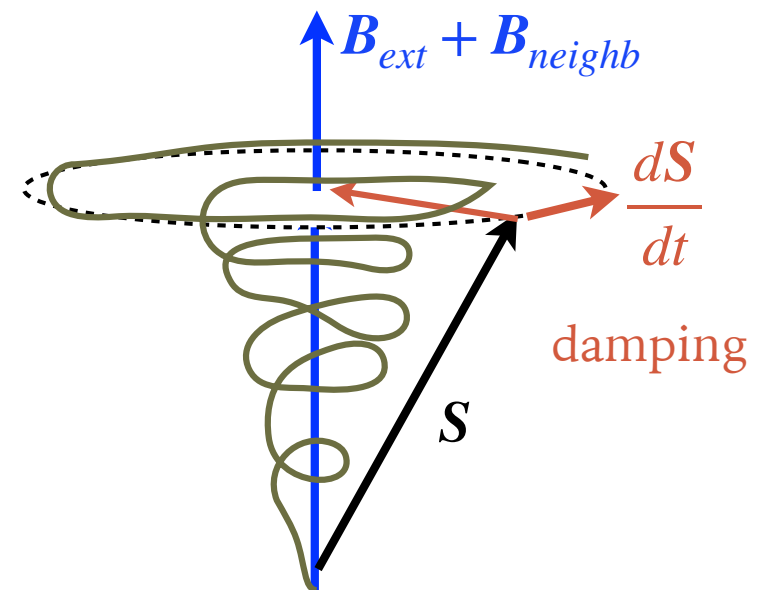
LLG

$$\frac{d\mathbf{S}}{dt} = \mathbf{S} \times \left(\gamma \mathbf{B}_{ext} + \gamma \mathbf{B}_{neighb} + \gamma \mathbf{b}_{th} - \gamma \eta \frac{d\mathbf{S}}{dt} \right)$$

gyromagnetic
ratio γ
(can be +/-)

externally
applied field
+ field from
interaction with
neighbours
(exchange)

stochastic
thermal field



KEY MESSAGE

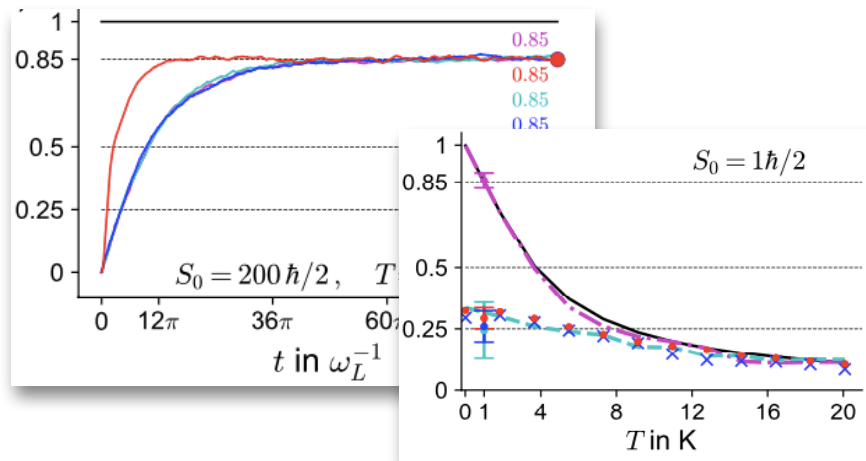


(different from Caldeira-Leggett and spin boson models)

Based on system+bath Hamiltonian, derived general three-dimensional spin dynamics equation

$$\frac{d\hat{\mathbf{S}}^{(n)}(t)}{dt} = \hat{\mathbf{S}}^{(n)}(t) \times \left[\gamma \mathbf{B}_{\text{ext}} + \sum_{m \neq n} \bar{\mathcal{J}}^{(nm)} \cdot \hat{\mathbf{S}}^{(m)}(t) + \gamma \hat{\mathbf{b}}^{(n)}(t) + \gamma^2 \int_{-\infty}^t dt' \mathcal{K}^{(n)}(t-t') \cdot \hat{\mathbf{S}}^{(n)}(t') \right], \quad (11)$$

Beyond LLG dynamics: faster magnetisation equilibration and non-Gibbs steady state



A quantum thermodynamic model directly relevant to current magnetism technology (hard drives)

Versatile three-dimensional quantum spin dynamics equation with guaranteed fluctuation-dissipation link
Anders, Sait, Horsley, arxiv 2009.00600v1 (2020)

OUTLINE

- System + bath Hamiltonian
- General spin dynamics equation
- LLG and Lorentzian coupling
- Short-time dynamics
- Equilibration
- Steady state magnetisation
- Conclusions and open questions

SYSTEM+BATH HAMILTONIAN

$$\hat{H} = \hat{H}_S + \hat{H}_R + \hat{V}_{\text{int}}$$

free Larmor frequency

$$\omega_L = |\gamma \mathbf{B}_{\text{ext}}|$$

system Hamiltonian

$$\hat{H}_S = -\gamma \sum_n \hat{\mathbf{S}}^{(n)} \cdot \mathbf{B}_{\text{ext}} - \frac{1}{2} \sum_{n \neq m} \hat{\mathbf{S}}^{(n)} \cdot \mathcal{J}^{(nm)} \cdot \hat{\mathbf{S}}^{(m)}$$

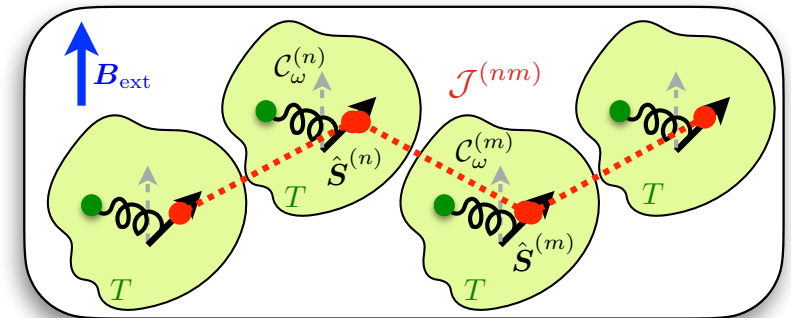
spin vector operator

at lattice site n $[\hat{S}_j^{(n)}, \hat{S}_k^{(m)}] = i\hbar \delta_{mn} \sum_l \epsilon_{jkl} \hat{S}_l^{(n)}$

bath Hamiltonian

$$\hat{H}_R = \frac{1}{2} \sum_n \int_0^\infty d\omega \left[\left(\hat{\mathbf{P}}_\omega^{(n)} \right)^2 + \omega^2 \left(\hat{\mathbf{X}}_\omega^{(n)} \right)^2 \right]$$

continuous bath of (3D) harmonic oscillators
(one for each site n)



Huttner, Barnett,
PRA 46 4306 (1992)

SYSTEM+BATH HAMILTONIAN

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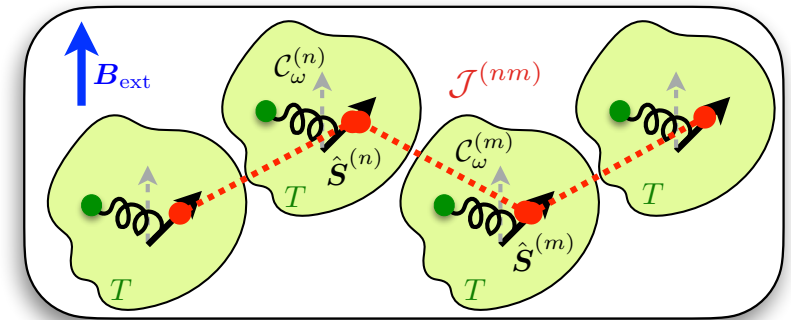
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continuous bath of (3D) harmonic oscillators
(one for each site n)

linear interaction

$$\hat{V}_{\text{int}} = -\gamma \sum_n \hat{\mathbf{S}}^{(n)} \cdot \int_0^\infty d\omega \mathcal{C}_\omega^{(n)} \cdot \hat{\mathbf{X}}_\omega^{(n)}$$

coupling function (tensor)
(one for each site n)

Huttner, Barnett,
PRA 46 4306 (1992)

GENERAL SPIN DYNAMICS EQUATION

... Heisenberg picture equation for the spin vector operator at site n :

$$\frac{d\hat{\mathbf{S}}^{(n)}(t)}{dt} = \hat{\mathbf{S}}^{(n)}(t) \times \left[\gamma \mathbf{B}_{\text{ext}} + \sum_{m \neq n} \mathcal{J}^{(nm)} \cdot \hat{\mathbf{S}}^{(m)}(t) + \gamma \hat{\mathbf{b}}^{(n)}(t) + \gamma^2 \int_{-\infty}^t dt' \mathcal{K}^{(n)}(t-t') \cdot \hat{\mathbf{S}}^{(n)}(t') \right]$$

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stochastic noise field: $\hat{\mathbf{b}}^{(n)}(t) = \int_0^\infty d\omega \sqrt{\frac{\hbar}{2\omega}} \mathcal{C}_\omega^{(n)} \left(\hat{\mathbf{a}}_\omega^{(n)} e^{-i\omega t} + \hat{\mathbf{a}}_\omega^{(n)\dagger} e^{+i\omega t} \right)$

memory kernel: $\mathcal{K}^{(n)}(\tau) = \Theta(\tau) \int_0^\infty d\omega \frac{\mathcal{C}_\omega^{(n)} \cdot \mathcal{C}_\omega^{(n)\dagger}}{\omega} \sin(\omega\tau)$

b and K fulfil a fluctuation dissipation relation

FLUCTUATION-DISSIPATION RELATION

Kubo (1966)

FDR

$$\tilde{P}_{jk}^{(nn)}(\omega) = \hbar \operatorname{Im}[\tilde{\mathcal{K}}_{jk}^{(n)}(\omega)] \coth \frac{\beta \hbar \omega}{2}$$

noise present
at 0K

fluctuations
in noise ($\hat{\mathbf{b}}$)

dissipation
(damping)

quantum
(thermal) noise

approximation
for high T /low ω :

$$k_B T \gg \frac{\hbar \omega}{2}$$

$$\tilde{P}_{jk}^{(nn)}(\omega) = \hbar \operatorname{Im}[\tilde{\mathcal{K}}_{jk}^{(n)}(\omega)] \frac{2}{\beta \hbar \omega}$$

no 0K noise

(classical)
white noise

FLUCTUATION-DISSIPATION RELATION

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noise present at 0K

fluctuations in noise ($\hat{\mathbf{b}}$)

dissipation (damping)

quantum (thermal) noise

for any coupling function $C_{\omega}^{(n)}$

approximation for high T /low ω :

$$k_B T \gg \frac{\hbar \omega}{2}$$

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no 0K noise

(classical) white noise

LLG AND LORENTZIAN COUPLING

choose for all n

linear coupling in ω :

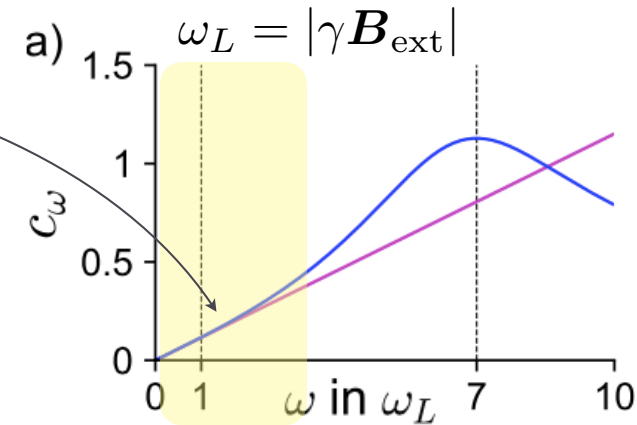
$$c_{\omega}^{\text{LLG}} = \sqrt{\frac{2\eta \cos(\omega\epsilon^+)}{\pi}} \frac{\omega}{\omega_L} \quad (\text{Ohmic})$$

(unit-free) $\epsilon^+ = 0^\downarrow$

get instantaneous
memory kernel:

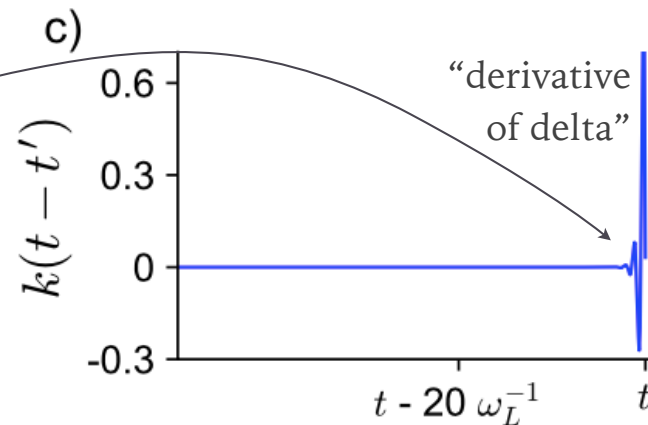
$$k^{\text{LLG}}(t - t') = \frac{\eta}{\omega_L^2} \partial_{t'} \delta(t^- - t')$$

(unit-free) $t^- = t - \epsilon^+$



LLG

linear



no
memory

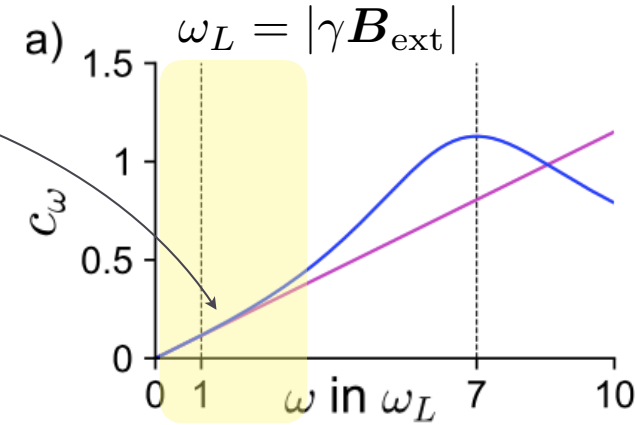
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LLG

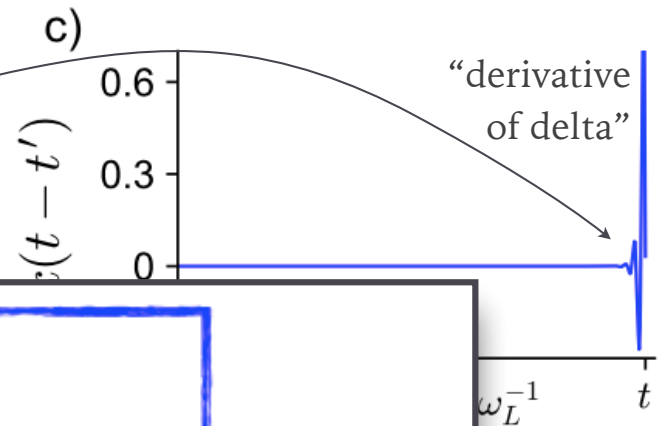
linear

get instantaneous memory kernel:

$$t^- = t - \epsilon^+$$

$$k^{\text{LLG}}(t - t') = \frac{\eta}{\omega_L^2} \partial_{t'} \delta(t^- - t')$$

(unit-free)



no memory

$$\frac{d\hat{S}^{(n)}(t)}{dt} = \hat{S}^{(n)}(t) \times \left[\gamma \mathbf{B}_{\text{ext}} + \sum_{m \neq n} \mathcal{J}^{(nm)} \cdot \hat{S}^{(m)}(t) + \gamma \hat{\mathbf{b}}^{(n)}(t) + \gamma^2 \int_{-\infty}^t dt' \mathcal{K}^{(n)}(t-t') \cdot \hat{S}^{(n)}(t') \right]$$

get: $\int_{-\infty}^t dt' \mathcal{K}^{(n)}(t-t') \cdot \hat{S}^{(n)}(t') \propto \int_{-\infty}^t dt' \eta \partial_{t'} \delta(t^- - t') \mathbf{S}(t') \propto -\eta \partial_t \mathbf{S}(t)$

recover:

LLG $\frac{d\mathbf{S}}{dt} = \mathbf{S} \times \left(\gamma \mathbf{B}_{\text{ext}} + \gamma \mathbf{B}_{\text{neighb}} + \gamma \mathbf{b}_{\text{th}} - \gamma \eta \frac{d\mathbf{S}}{dt} \right)$ ✓

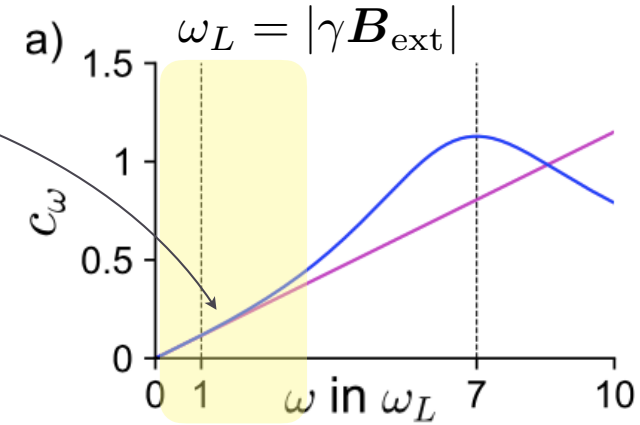
LLG AND LORENTZIAN COUPLING

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linear coupling in ω :

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LLG

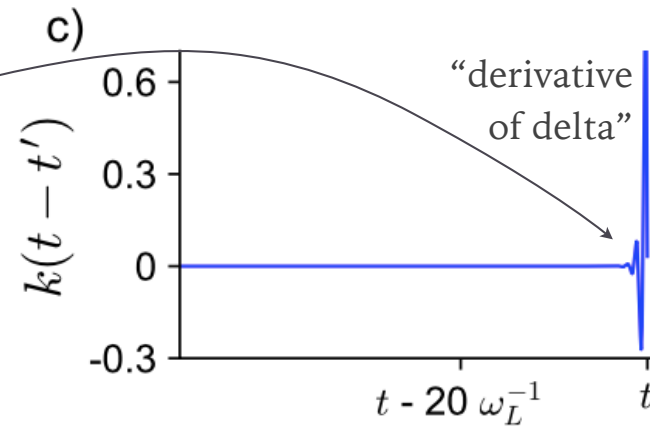
linear

get instantaneous memory kernel:

$$t^- = t - \epsilon^+$$

$$k^{\text{LLG}}(t - t') = \frac{\eta}{\omega_L^2} \partial_{t'} \delta(t^- - t')$$

(unit-free)



no memory

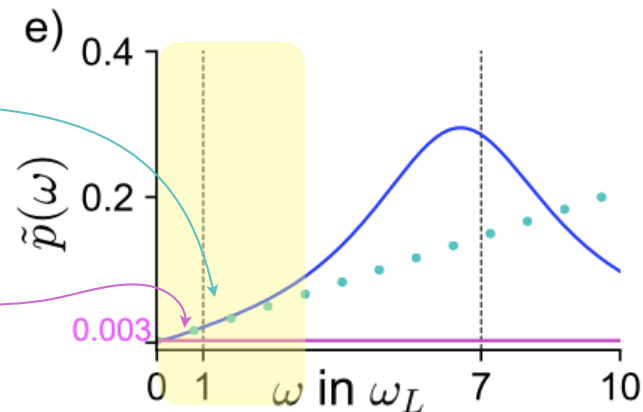
and monotonic power spectrum:

$$\tilde{p}_{\text{qu}}^{\text{LLG}}(\omega) = \frac{\eta \hbar \omega}{\hbar \omega_L} \coth\left(\frac{\beta \hbar \omega}{2}\right)$$

(unit-free)

$$k_B T \gg \hbar \omega : \tilde{p}_{\text{cl}}^{\text{LLG}}(\omega) = \eta \frac{2k_B T}{\hbar \omega_L}$$

“white noise”

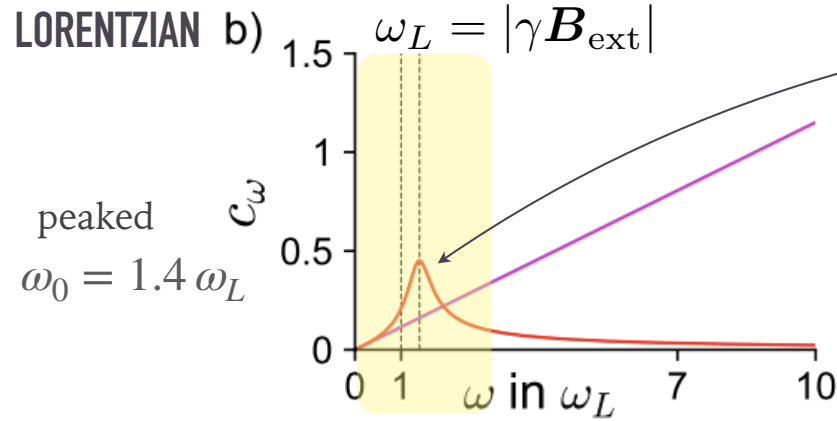


$T = 1K$

linear

flat

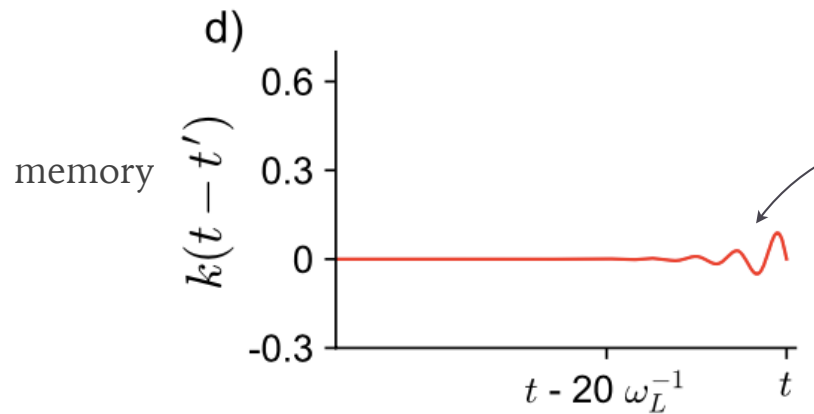
LLG AND LORENTZIAN COUPLING



choose for all n

Lorentzian coupling in ω :

$$c_{\omega}^{\text{Lor}} = \sqrt{\frac{2\alpha\Gamma}{\pi} \frac{\omega^2}{(\omega_0^2 - \omega^2)^2 + \omega^2\Gamma^2}}$$



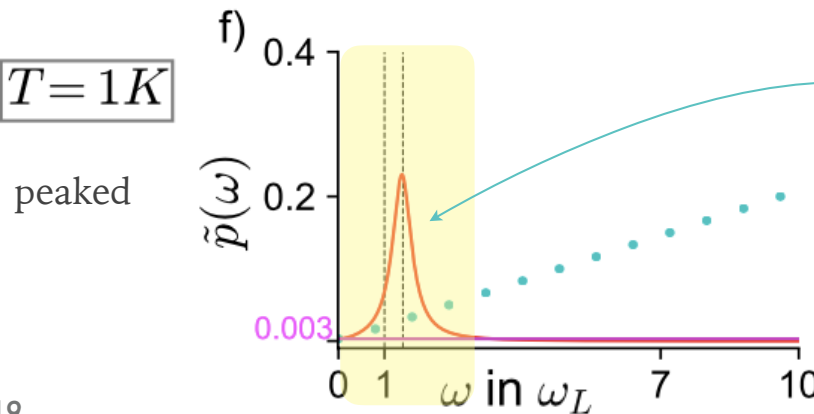
get oscillating and decaying
memory kernel:

$$k^{\text{Lor}}(\tau) = \Theta(\tau) \alpha e^{-\Gamma\tau/2} \frac{\sin(\omega_1\tau)}{\omega_1}$$

$$\tau = t - t'$$

$$\omega_1 = \sqrt{\omega_0^2 - \Gamma^2/4}$$

$T = 1K$



and peaked power
spectrum:

$$\tilde{p}_{\text{qu}}^{\text{Lor}}(\omega) = \frac{\alpha\Gamma\omega_L\omega}{(\omega_0^2 - \omega^2)^2 + \omega^2\Gamma^2} \coth \frac{\beta\hbar\omega}{2}$$

SHORT TIME DYNAMICS

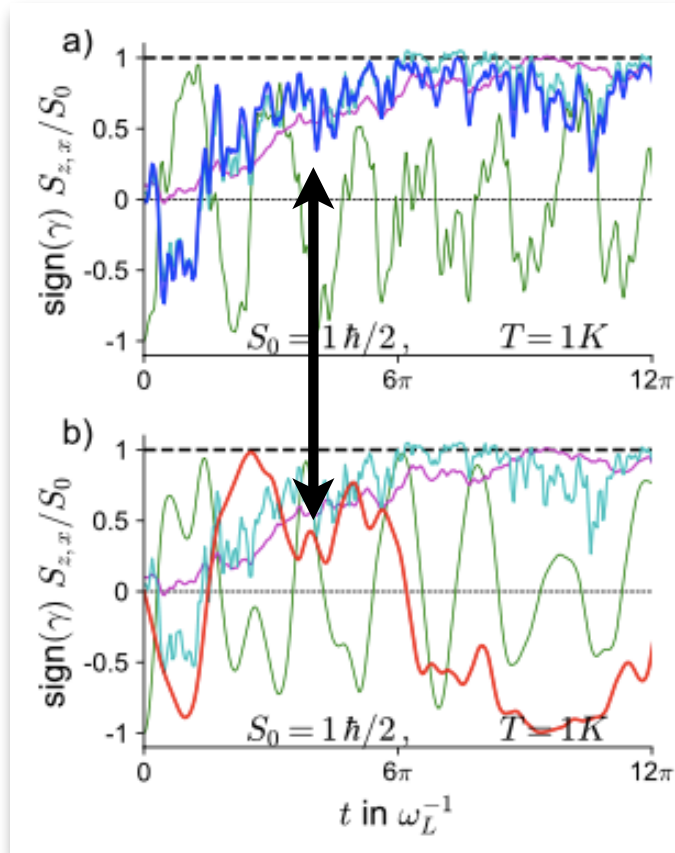
stochastic dynamics of a single classical spin vector (no exchange)

S_z in time for LLG-like and Lorentzian coupling with same linear η :

small spin $|S| = 1 \frac{\hbar}{2}$
at $T = 1K$

S_z and S_x for LLG-like coupling

S_z and S_x for Lorentzian coupling



memory effects make dynamics completely different!

(offset to see deviations)
(same in upper+lower panels)

S_z in linear LLG approx. with quantum noise

SHORT TIME DYNAMICS

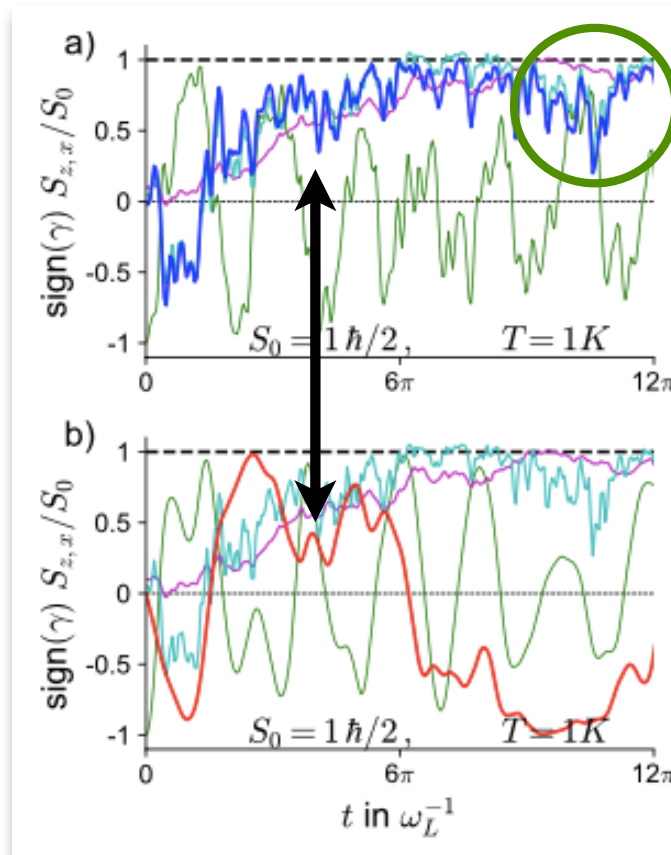
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quantum noise makes even LLG-like dynamics different

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(offset to see deviations)
(same in upper+lower panels)

S_z in linear LLG approx. with quantum noise

S_z in linear LLG approx. with white noise (double offset to see deviations)
(same in all four subpanels)

SHORT TIME DYNAMICS

stochastic dynamics of a single classical spin vector (no exchange)

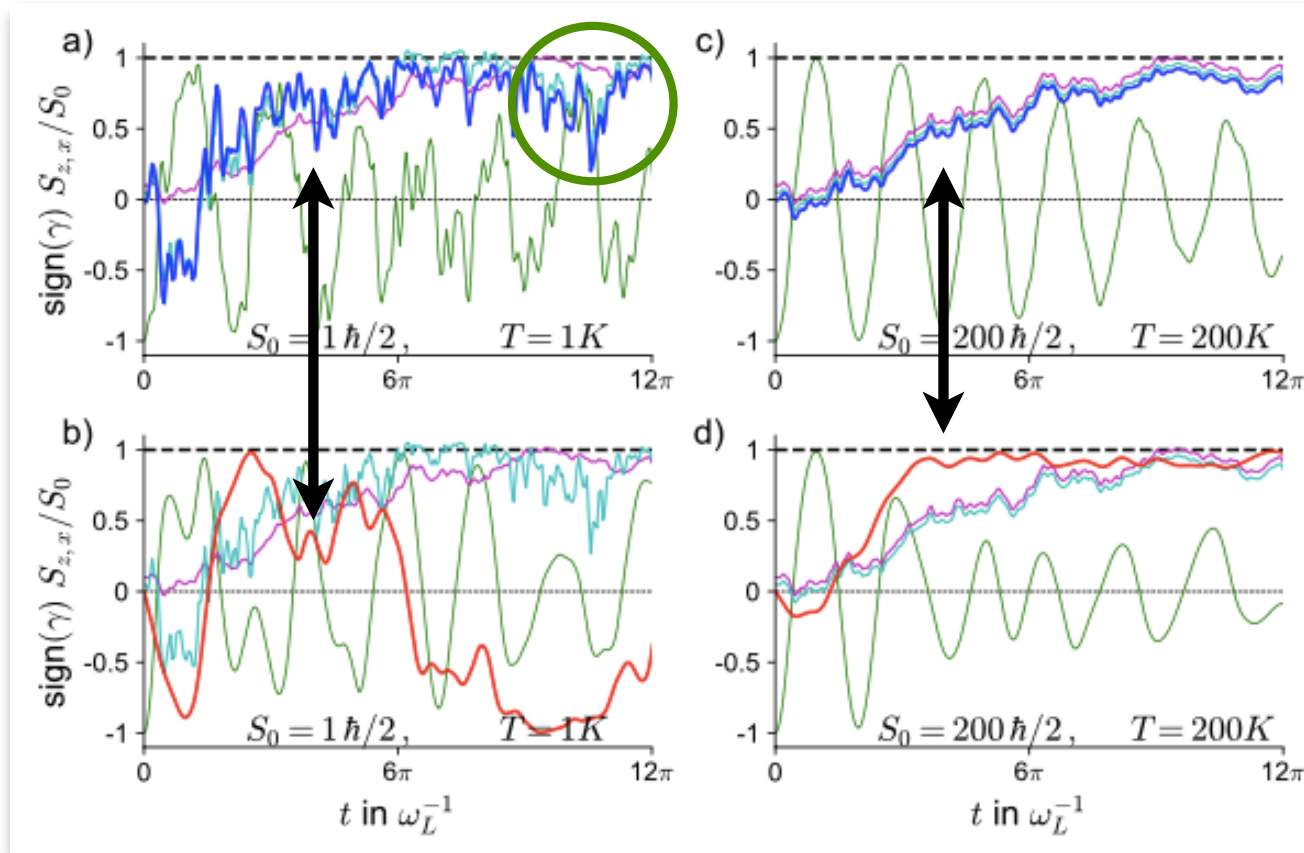
S_z in time for LLG-like and Lorentzian coupling with same linear η :

small spin $|S| = 1 \frac{\hbar}{2}$ at $T=1K$

mesoscopic spin $|S| = 200 \frac{\hbar}{2}$ at $T=200K$

S_z and S_x for LLG-like coupling

S_z and S_x for Lorentzian coupling



quantum noise makes even LLG-like dynamics different

memory effects make dynamics completely different!

(offset to see deviations)
(same in upper+lower panels)

S_z in linear LLG approx. with quantum noise

S_z in linear LLG approx. with white noise (double offset to see deviations) (same in all four subpanels)

OUTLINE

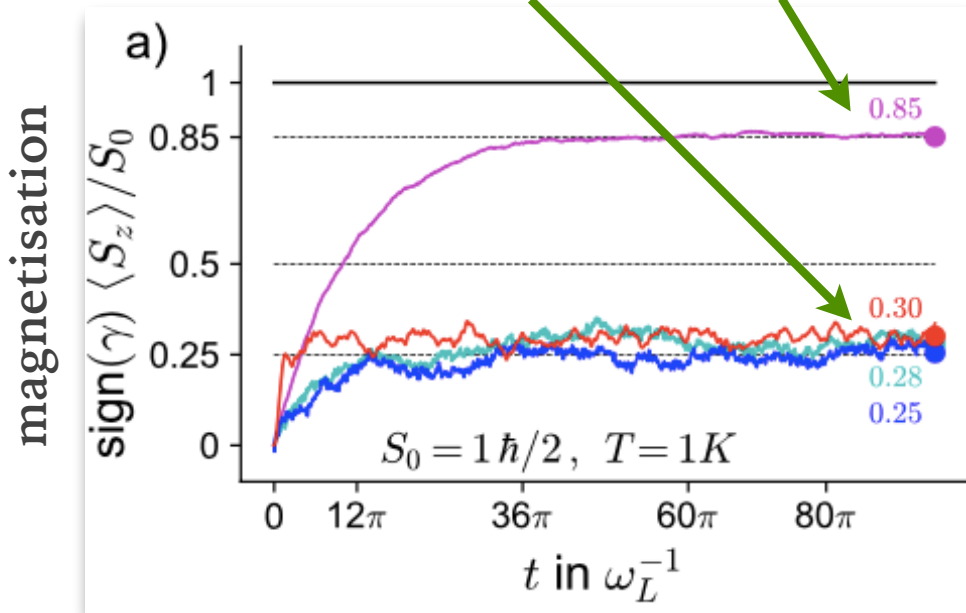
- System + bath Hamiltonian
- General spin dynamics equation
- LLG and Lorentzian coupling
- Short-time dynamics
- Equilibration
- Steady state magnetisation
- Conclusions and open questions

EQUILIBRATION

ensemble average dynamics

small spin

quantum noise white noise



LLG-like coupling

Lorentzian coupling

linear approx. with quantum noise

or linear approx. with white noise

EQUILIBRATION

ensemble average dynamics

small spin

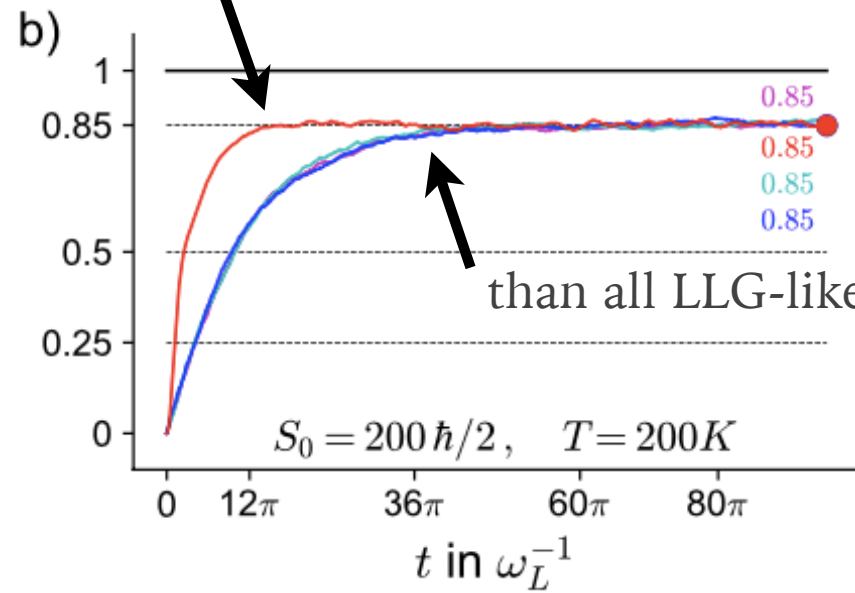
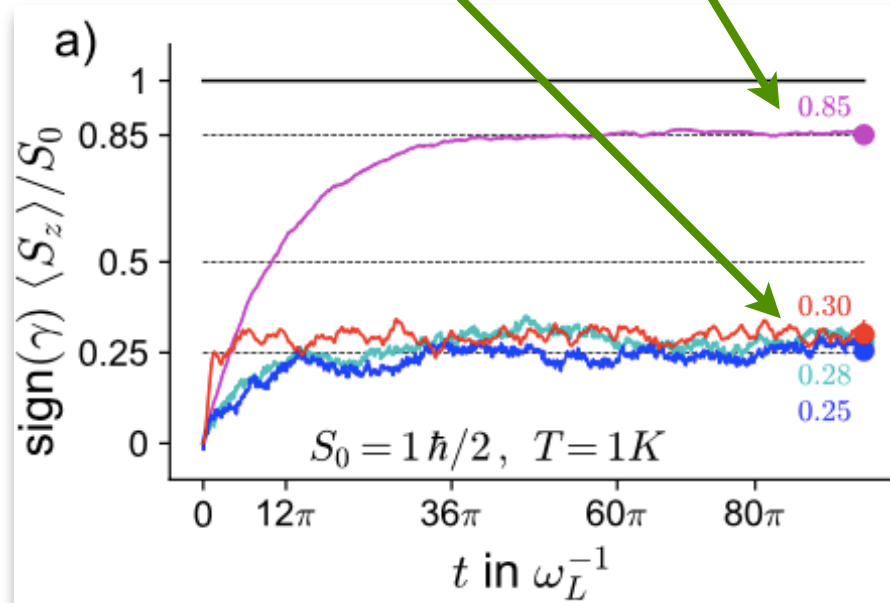
mesoscopic spin

quantum noise

white noise

Lorentzian: quicker equilibration

magnetisation



LLG-like coupling

Lorentzian coupling

linear approx. with quantum noise

or linear approx. with white noise

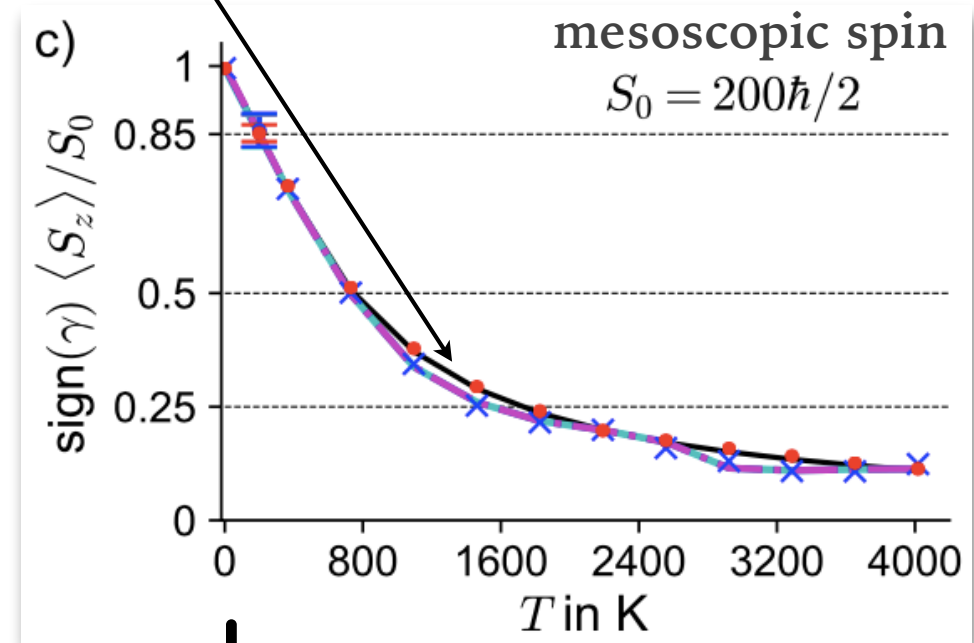
STEADY STATE MAGNETISATION

$$\langle S_z \rangle = \int_{-S_0}^{+S_0} dS_z S_z \frac{e^{-\beta(-\gamma S_z B_{ext})}}{Z}$$



$$\frac{\langle S_z \rangle}{S_0} = \text{sign}(\gamma) \left(\coth \left(\frac{S_0 \omega_L}{k_B T} \right) - \frac{k_B T}{S_0 \omega_L} \right)$$

where temperature
is scaled by $S_0 = |S|$



LLG-like coupling

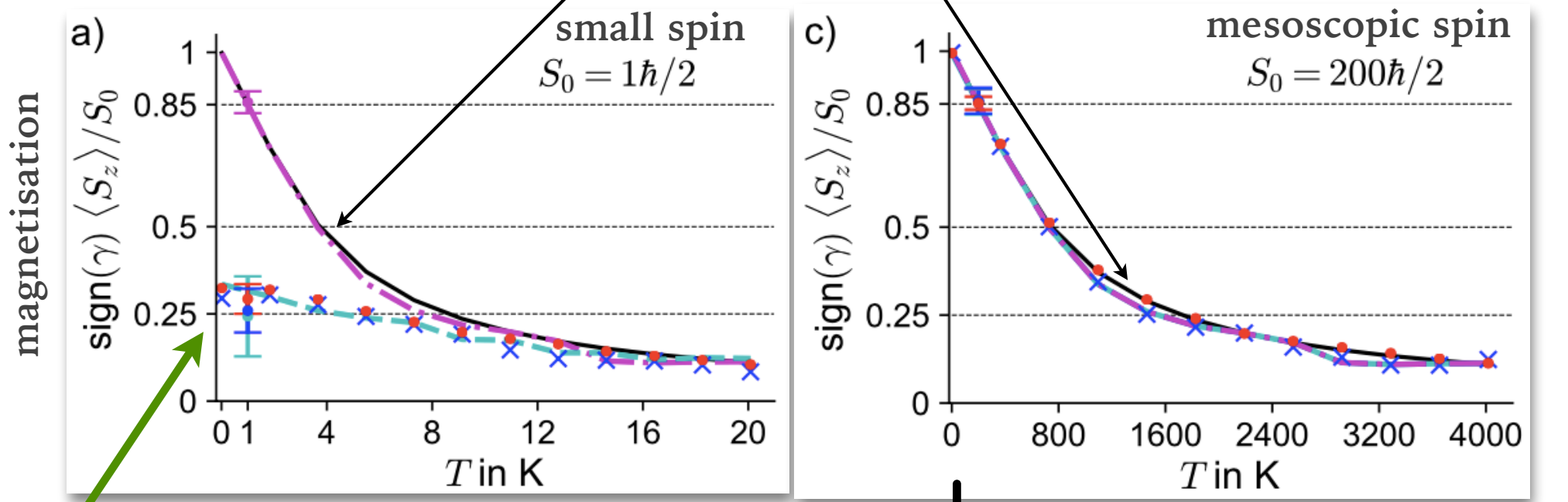
Lorentzian coupling

linear approx. with quantum noise
or linear approx. with white noise

STEADY STATE MAGNETISATION

$$\langle S_z \rangle = \int_{-S_0}^{+S_0} dS_z S_z \frac{e^{-\beta(-\gamma S_z B_{ext})}}{Z} \quad \longrightarrow \quad \frac{\langle S_z \rangle}{S_0} = \text{sign}(\gamma) \left(\coth \left(\frac{S_0 \omega_L}{k_B T} \right) - \frac{k_B T}{S_0 \omega_L} \right)$$

where temperature is scaled by $S_0 = |S|$



reduced magnetisation due to quantum noise even at 0K

LLG-like coupling
 Lorentzian coupling

linear approx. with quantum noise
 or linear approx. with white noise

OUTLINE

- System + bath Hamiltonian
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Versatile three-dimensional quantum spin dynamics equation with guaranteed fluctuation-dissipation link
Anders, Sait, Horsley, arxiv 2009.00600v1 (2020)



... beyond Caldeira-Leggett and Spin-boson

based on system + bath Hamiltonian, have derived three-dimensional general spin dynamics equation

$$\frac{d\hat{\mathbf{S}}^{(n)}(t)}{dt} = \hat{\mathbf{S}}^{(n)}(t) \times \left[\gamma \mathbf{B}_{\text{ext}} + \sum_{m \neq n} \bar{\mathcal{J}}^{(nm)} \cdot \hat{\mathbf{S}}^{(m)}(t) + \gamma \hat{\mathbf{b}}^{(n)}(t) + \gamma^2 \int_{-\infty}^t dt' \mathcal{K}^{(n)}(t-t') \cdot \hat{\mathbf{S}}^{(n)}(t') \right], \quad (11)$$

PHYSICAL REVIEW LETTERS **121**, 040401 (2018)

Rotational Friction and Diffusion of Quantum Rotors

Benjamin A. Stickler,^{*} Björn Schrämski,[†] and Klaus Hornberger

Research Article

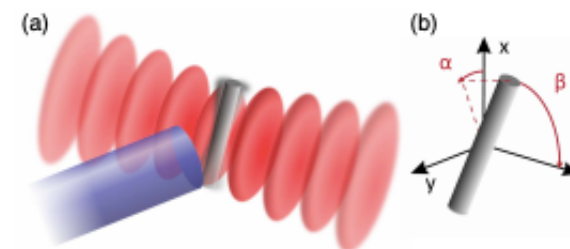
Vol. 4, No. 3 / March 2017 / Optica 356

optica

Full rotational control of levitated silicon nanorods

STEFAN KUHN,^{1,*†} ALON KOSLOFF,^{2,†} BENJAMIN A. STICKLER,³ FERNANDO PATOLSKY,² KLAUS HORNBERGER,³ MARKUS ARNDT,¹ AND JAMES MILLEN¹

applicable for 3D rotational Brownian motion





... beyond Caldeira-Leggett and Spin-boson

based on system + bath Hamiltonian, have derived **three-dimensional** general spin dynamics equation

$$\frac{d\hat{\mathbf{S}}^{(n)}(t)}{dt} = \hat{\mathbf{S}}^{(n)}(t) \times \left[\gamma \mathbf{B}_{\text{ext}} + \sum_{m \neq n} \bar{\mathcal{J}}^{(nm)} \cdot \hat{\mathbf{S}}^{(m)}(t) + \gamma \hat{\mathbf{b}}^{(n)}(t) + \gamma^2 \int_{-\infty}^t dt' \mathcal{K}^{(n)}(t-t') \cdot \hat{\mathbf{S}}^{(n)}(t') \right], \quad (11)$$

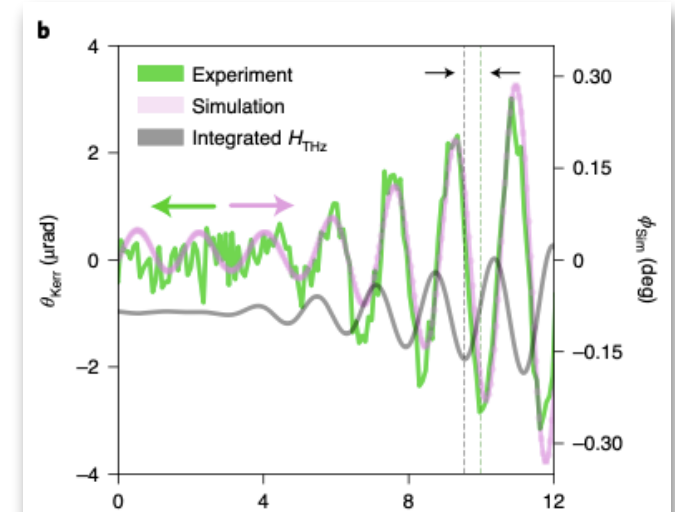
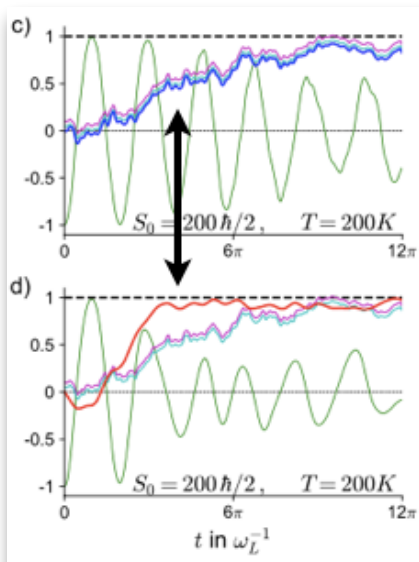
magnetism:

For linear coupling function (Ohmic), recover widely used LLG equation.

Lorentzian couplings allow to systematically study how memory kernels affect the short/long time dynamics.

(with guaranteed FDR)

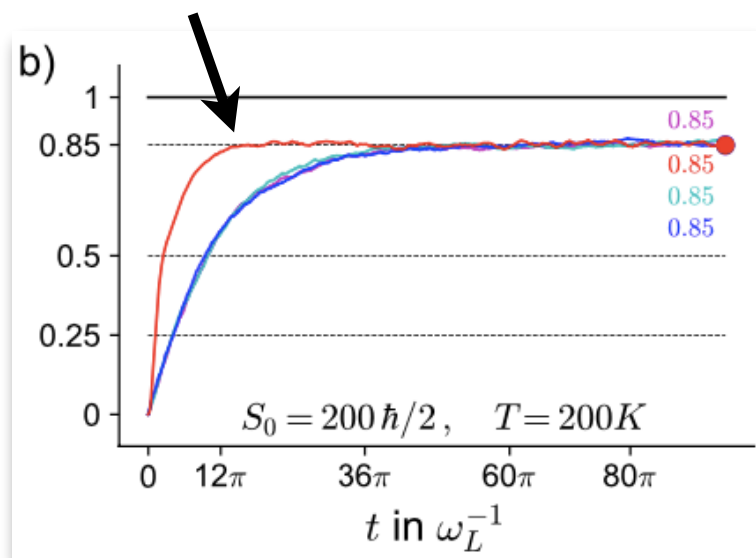
simulation with memory kernel fits very recent measurements:



Neeraj, et al., Bonetti, Nat. Phys. (2020)



Memory effects lead to faster magnetisation equilibration.



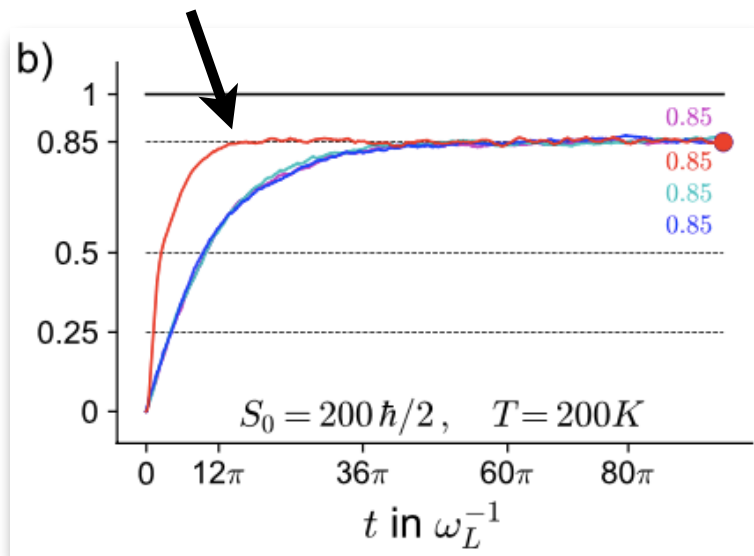
Predictions of the short time dynamics and equilibration to steady state strongly depend on the parameters of the Lorentzian coupling.

What is the precise link between non-delta memory kernel (non-Markovian dynamics) and the equilibration time?

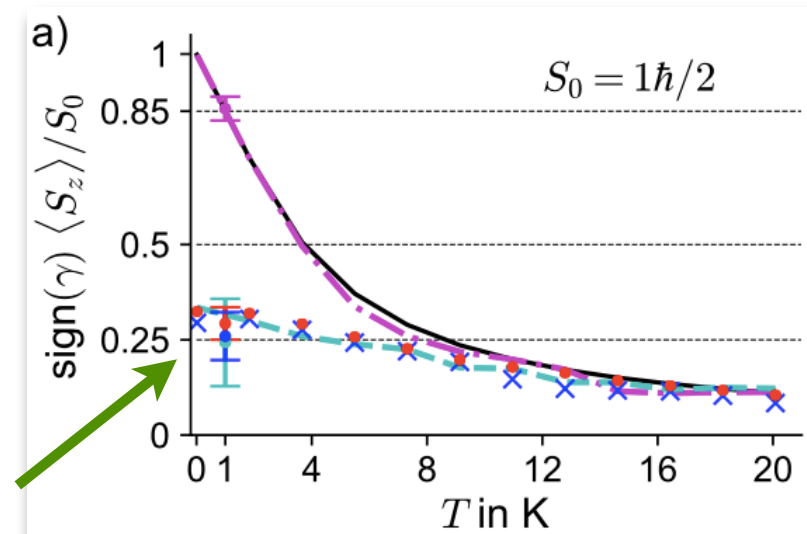
(please point out references)



Memory effects lead to faster magnetisation equilibration.



Non-Gibbs steady state due to quantum noise



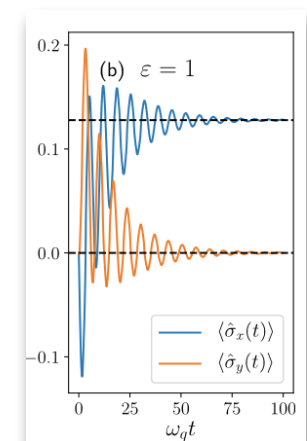
What is the precise link between non-delta memory kernel (non-Markovian dynamics) and the equilibration time?

(please point out references)

strong coupling thermodyn?

Link to Hamiltonian of mean force?

Purkayastha, et al., Goold, npj Quantum Information (2020)



- timescale of equilibration and steady state?
 - non-isotropic coupling tensors
 - 3D vs 1D effects
 - thin magnetic layers
- $$\hat{V}_{\text{int}} = -\gamma \sum_n \hat{\mathbf{S}}^{(n)} \cdot \int_0^\infty d\omega \mathcal{C}_\omega^{(n)} \cdot \hat{\mathbf{X}}_\omega^{(n)}$$
- differences between quantum and classical spin dynamics
 - bath modes at different temperatures
 - heat transport
 - electron/phonon baths
 - common baths? see Camille's talk
 - including driving with EM fields/light in equations of motion
 - replace LLG in micromagnetic/atomistic simulations with general spin dynamics equation



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Thank you!

